

Distributed Binary Quantizers for Communication Constrained Large-scale Sensor Networks

Ying Lin and Biao Chen
Dept. of EECS
Syracuse University
Syracuse, NY 13244, U.S.A.
ylin20 {bichen}@ecs.syr.edu

Peter Willett
Dept. of ECE
University of Connecticut
Storrs, Connecticut, U.S.A.
willett@engr.uconn.edu

Bruce Suter
AFRL/IFGC
525 Brooks Rd
Rome, NY, U.S.A.
suterb@rl.af.mil

Abstract - We consider in this paper local sensor quantizer design for large-scale bandwidth and/or energy constrained wireless sensor networks (WSNs) operating in fading channels. In particular, under the Neyman-Pearson framework, we address the design of binary local sensor quantizers for a binary hypothesis problem in the asymptotic regime where the number of sensors is large. Motivated by the sensor censoring idea for reduced communication rate, each sensor either transmits ‘1’ to a fusion center or remains silent. By adopting energy detector as the fusion rule, we develop a procedure to obtain local sensor threshold that maximizes the Kullback-Leibler distance of the distributions of the fusion statistic under the two hypotheses. The proposed quantizer design is well suited for the emerging large scale resource-constrained WSNs applications. Numerical results based on Gaussian and exponential observations are presented to demonstrate the design procedure.

Keywords: Wireless sensor networks, distributed detection, asymptotic regime, censoring sensors.

1 Introduction

The design of local quantizer for distributed detection under communication rate constraint has been studied over the past decades (see, e.g., [1–7]). For example, the sensor censoring idea, was proposed by Rago *et al* in 1996 [1] in the context of decentralized detection for reduced communication rate. With censored sensors, only the sensor with informative observation, measured by its local likelihood ratio (LR) value, sends its LR value to the fusion center. Under Neyman-Pearson framework, an extreme censoring scheme with an on/off local sensor signaling structure has been considered in [3] in the context of studying locally optimum distributed detection, where if the local LR exceeds certain threshold, then a single bit of information is sent; otherwise, the sensor keeps silent. More recently, under the Bayesian framework, the design of optimal local thresholds for distributed detection under such on/off signaling structure was studied in [7].

In this paper, we consider the Neyman-Pearson framework and address the optimal binary local sen-

sor quantizer design for a binary hypothesis testing problem. In particular, we study the asymptotic case where the number of sensors is large. Using an energy detector as the fusion statistic at the fusion center, we propose a procedure to determine the optimal local threshold through maximizing the Kullback-Leibler (KL) distance of the distributions of the fusion statistic under the two hypotheses. Specifically, we develop efficient algorithms to facilitate the determination of the optimal thresholds for various scenarios, catering to different constraints.

The organization of the paper is as follows. In the next section, we introduce the system model and formulate the problem. In Section 3, we pose the design problem as maximizing the KL distance while subject to rate constraints at local sensors and present algorithms to obtain the optimal thresholds numerically. Design examples under Gaussian and exponential distributions are provided in Section 4 to illustrate the design procedure. We finally conclude in Section 5.

2 Problem formulation

Fig. 1 depicts a canonical parallel fusion structure in the presence of non-ideal channels. All the channels are assumed independent Rayleigh fading channels and corrupted by independent identically distributed (i.i.d.) complex white Gaussian noises. Specifically, we use $h_k e^{j\phi_k}$ and n_k to denote the channel coefficient and channel noise of the k^{th} channel. The quantity h_k follows a Rayleigh distribution with pdf of $P(h_k) = 2h_k e^{-h_k^2}$, $\phi_k \in U[0, 2\pi]$ is the phase due to the transmission, and n_k is a zero mean complex Gaussian noise whose real and imaginary parts are independent of each other and have equal variance σ^2 .

Assume the local observations X_k , $k = 1, 2, \dots, K$, are independent across sensors conditioned on the hypotheses, i.e., for $i = 0, 1$,

$$P(X_1, \dots, X_K | H_i) = \prod_{k=1}^K P(X_k | H_i),$$

where K is the total number of sensors and H_0 and H_1 represent the two hypotheses. We further assume that each local sensor makes a binary decision u_k based on

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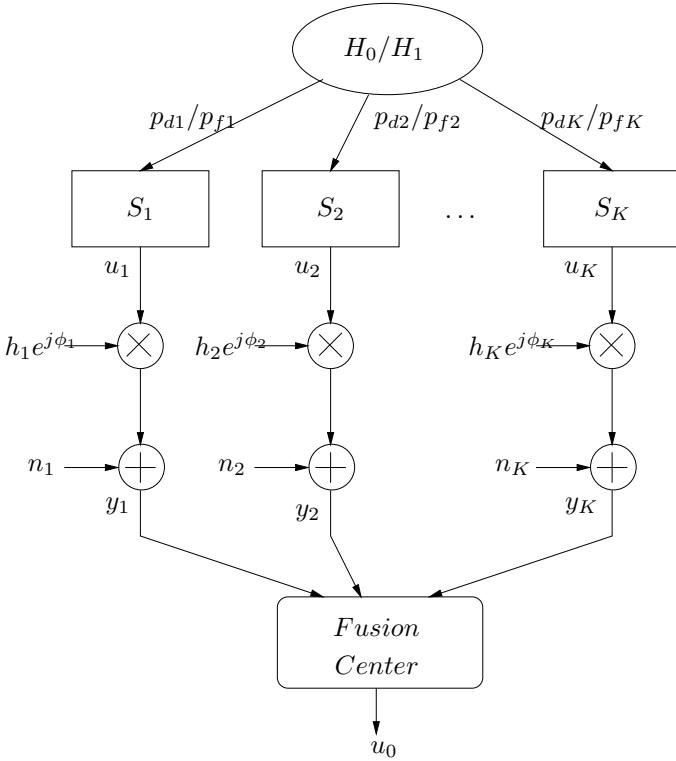


Figure 1: Parallel fusion model in the presence of Rayleigh fading and noisy channels between the local sensors and the fusion center

its observation X_k :

$$u_k = \gamma_k(X_k) \in \{1, 0\}$$

where $\gamma_k(\cdot)$ stands for the local decision rule at the k^{th} sensor. If $u_k = 1$, sensor k will send u_k to the fusion center through a fading channel; Otherwise, the sensor remains silent, i.e., no transmission is needed.

Denote ρ as the communication constraint on each sensor. In this work, we assume all local sensors have the same communication constraint. Under the Neyman-Pearson framework, following the similar spirit as in [1], we define the communication rate R_k for the k^{th} sensor as the probability of false alarm rate at the sensor. Let p_{dk} and p_{fk} denote the detection probability and the false alarm rate at the k^{th} local sensor, the communication constraint requires the following condition to be satisfied:

$$R_k = p_{fk} \leq \rho$$

for $k = 1, \dots, K$. We note that this rate constraint is different from the constraint on the false alarm rate under the Neyman-Pearson test: the size of the test for a distributed detection system is defined for the fusion center whereas here we constrain the size of the local sensors.

At the fusion center, based on the received channel outputs y_1, \dots, y_K , a final decision u_0 on which hypothesis is true is made by implementing an optimal fusion rule γ_0 . That is,

$$u_0 = \gamma_0(y_1, \dots, y_K)$$

The goal in this paper is to obtain the optimal local decision rules (i.e., find the optimal local sensor quantization thresholds) that achieves optimal detection performance under the communication constraint. In particular, under the Neyman-Pearson framework, we consider the asymptotic case where the number of sensors is large. The design rationale and our main results are provided in the next section.

3 Asymptotic Regime

Throughout this paper, we consider K , the number of local sensors, to be large.

We assume all sensors adopt likelihood ratio (LR) test as the local decision rule with the same threshold, denoted by τ . This is motivated by the classical result [8] where it was shown that identical decision rule is asymptotically optimum for binary hypothesis testing with identically distributed observations. Thus, local sensors have identical local performance indexes, which we denote by $p_f = p_{fk}$ and $p_d = p_{dk}$. Moreover, the rate constraints for all sensors are the same, i.e., $R = p_f \leq \rho$.

Given the system model depicted in Fig. 1, the received signals at the fusion sensor can be expressed as, for $k = 1, 2, \dots, K$,

$$y_k = \begin{cases} n_k & \text{if } u_k = 0 \\ h_k e^{j\phi_k} + n_k & \text{if } u_k = 1 \end{cases} \quad (1)$$

In the current setup, we assume that only channel fading statistics are available at the design stage. Therefore, for incoherent detection, we adopt an energy detector as the fusion statistic, i.e.,

$$\Lambda = \frac{1}{K} \sum_{k=1}^K |y_k|^2 = \frac{1}{K} \sum_{k=1}^K z_k$$

where $z_k \triangleq |y_k|^2$ and is independent of one another for $k = 1, \dots, K$, given the independence assumptions on the channels and on channel noises. It is shown in [5, 6] that the energy detector is a good approximate of the optimal fusion rule at low signal-to-noise ratio regime. The conditional pdfs of z_k given the local decision u_k are given by:

$$\begin{aligned} P(z_k | u_k = 0) &= \frac{1}{2\sigma^2} e^{-\frac{z_k}{2\sigma^2}} \\ P(z_k | u_k = 1) &= \frac{1}{1+2\sigma^2} e^{-\frac{z_k}{1+2\sigma^2}} \end{aligned}$$

That is, given the local decisions u_k , z_k 's are i.i.d. and are exponentially distributed with respective mean values equal to $2\sigma^2$ and $1+2\sigma^2$.

Consider the number of sensors to be large, the Central Limit Theorem allows us to approximate the distributions of Λ as Gaussian distributions under both hypotheses. The following lemma describes the distributions of Λ specifically.

Lemma 1 *In the asymptotic regime, the conditional pdf of Λ given hypothesis H_j is $N(\mu_j, \sigma_j^2)$ for $j = 0, 1$.*

Specifically,

$$\begin{aligned}\mu_1 &= E[\Lambda|H_1] = p_d + 2\sigma^2 \\ \mu_0 &= E[\Lambda|H_0] = p_f + 2\sigma^2 \\ \sigma_1^2 &= \text{Var}[\Lambda|H_1] = f(p_d) \\ \sigma_0^2 &= \text{Var}[\Lambda|H_0] = f(p_f)\end{aligned}$$

where

$$f(x) \triangleq \frac{1}{K}[(2\sigma^2)^2 + 2(1+2\sigma^2)x - x^2] \quad (2)$$

Since p_d and p_f are functions of the local LR threshold τ , both means and variances of Λ under both hypotheses are also functions of τ .

The KL distance (relative entropy), defined as $D(P_0||P_1) = E_{p_0}(\log \frac{P_0}{P_1})$, between the two distributions under test is directly related to the detection performance in an asymptotic regime given $P_j = P(\Lambda|H_j)$, $j = 0, 1$. Stein's lemma [9] states that under the Neyman-Pearson framework the best achievable error exponent in the probability of error is given by the KL distance.

In our setup, for Gaussian distributions P_0 and P_1 , we can further simplify $D(P_0||P_1)$ as

$$D(P_0||P_1) = \log \frac{\sigma_1}{\sigma_0} + \frac{(\sigma_0^2 - \sigma_1^2) + (\mu_0 - \mu_1)}{2\sigma_1^2} \quad (3)$$

Throughout this paper, we use the natural logarithm in the KL distance expression, i.e., the measure unit of $D(P_0||P_1)$ is nats. It is shown in [9] that the KL distance is nonnegative. Clearly, the KL distance $D(P_0||P_1)$ is a function of local LR threshold τ . As such, the optimum local threshold τ can be determined by

$$\tau^* = \arg \max_{\tau} D(P_0||P_1) = \arg \max_{\tau} D(\tau) \quad (4)$$

To accommodate the communication rate constraints, we now pose the design problem as the following constrained maximization problem:

$$\begin{aligned}\max & \quad D(\tau) \\ \text{subject to} & \quad R = p_f \leq \rho\end{aligned} \quad (5)$$

This is a nonlinear optimization problem with inequality constraint [10]. In general, since $D(\tau)$ is not a convex function, we can not adopt the Kuhn-Tucker Theorem [10] directly to solve the above optimization problem.

The closed-form solution to the local sensor threshold may not be obtained directly. In the current work, however, since the rate function $R(\tau) = p_f(\tau)$ is a decreasing function as τ , we can devise an efficient algorithm to obtain the optimal τ numerically, as described below:

Algorithm

1. Obtain τ^* based on Eq. (4) where τ^* corresponds to the maximum point of the KL distance without any rate constraint.
2. Check if τ^* satisfies $R(\tau^*) \leq \rho$;

(a) If yes, stop, and the optimum solution $\tau_{opt} = \tau^*$;

(b) If not, go to 3).

3. Calculate $\tau^{(\rho)}$ by solving $R(\tau) - \rho = 0$. Since $R(\tau)$ monotonically decreases as τ increases, the optimum point must be in the region of $[\tau^{(\rho)}, \infty)$. Calculate the KL distance of the following three types of points within this region:

(a) the start point: $\tau^{(s)} = \tau^{(\rho)}$;

(b) the end point: $\tau^{(e)} = \infty$;

(c) the points $\tau^{(0,i)}$, $i = 1, 2, \dots, N_0$, which satisfy $\frac{\partial D(\tau)}{\partial \tau} = 0$. The quantity N_0 denotes the total number of such points.

4. Compare the KL distances associated with the points in 3). The optimum solution τ_{opt} is the threshold which corresponds to the largest KL distance among those obtained in 3).

In many cases, for example, for local observations with Gaussian and exponential distributions, the LR threshold τ can be translated directly to the local observation threshold, denoted as η . The rate function $R(\eta)$ is thus either a monotonically decreasing or increasing function of η . In the former case, the above algorithm is directly applicable with τ replaced by η . To deal with the latter case, i.e., the increasing rate function $R(\eta)$, we only need slightly modify the algorithm in step 3). Specifically,

- set the start point $\eta^{(s)} = \eta_s$, where η_s is the minimum value of η associated with the detection problem under consideration, e.g., in the Gaussian observations case, $\eta_s = -\infty$.

- set the end point $\eta^{(e)} = \eta^{(\rho)}$.

In the next section, we will present two design examples to find the optimal local thresholds using the proposed algorithms.

4 Design Examples

In this section, we demonstrate the design procedure described in the previous section through examples of Gaussian and exponential distributed local observations.

4.1 Gaussian Observations Case

In the case of the detection of a known signal in independent Gaussian noises, the observations at local sensors follow Gaussian distributions. Specifically, we assume that

$$\begin{aligned}H_0 : \quad X_k &= v_k \\ H_1 : \quad X_k &= s + v_k\end{aligned} \quad (6)$$

where s is the known signal, v_k , $k = 1, 2, \dots, K$, are i.i.d. white Gaussian noises with zero mean and

variance σ_v^2 . Without loss of generality, we assume $s = 1$ in the simulation.

Notice that for the Gaussian problem, the LR threshold τ can be directly translated to the thresholds for the local observations η . Then the false alarm rate p_f and the detection probability p_d at local sensors can be expressed as:

$$p_f = Q\left(\frac{\eta}{\sigma_v}\right) \quad (7)$$

$$p_d = Q\left(\frac{\eta - s}{\sigma_v}\right) \quad (8)$$

Thus, we further obtain

$$\begin{aligned} \mu_1 &= Q\left(\frac{\eta - s}{\sigma_v}\right) + 2\sigma^2 \\ \mu_0 &= Q\left(\frac{\eta}{\sigma_v}\right) + 2\sigma^2 \\ \sigma_1^2 &= f\left(Q\left(\frac{\eta - s}{\sigma_v}\right)\right) \\ \sigma_0^2 &= f\left(Q\left(\frac{\eta}{\sigma_v}\right)\right) \end{aligned}$$

where $f(x)$ is defined in Eq. (2) in the previous section.

Given the form of p_f in Eq. (8), we can show that the rate function $R = p_f$ is a monotonically decreasing function of η . Hence, the optimal threshold η can be obtained using the procedure described in the proposed algorithm. The simulation results under different rate constraint ρ are listed in Table 1 at the channel signal-to-noise ratio (SNR) = 0dB.

As seen from Table 1, the results obtained using the proposed algorithm and through exhaustive search match very well for different communication constraint ρ . Another interesting observation is: as ρ increases, the obtained threshold will eventually remain unchanged. This is not surprising: in such case, the optimal solution coincides with that of the unconstrained optimization.

To better understand this behavior, we provide a plot of the KL distance versus the rate constraint ρ in Fig. 2. Three different curves represent different channel SNRs. The tradeoff between the communication rate, the channel SNR and the system performance can be clearly seen from the figure. For small ρ , increasing ρ improves the KL distance yet as ρ becomes large, $D(P_0||P_1)$ eventually levels off in all cases. Moreover, the higher the channel SNR, the better the local quantization can do to improve the system performance.

4.2 Exponential Observations Case

Next we consider an example of exponential observations at local sensors. The conditional distributions under exponential observations assumption can be expressed as

$$f(X_k|H_0) = \beta_0 e^{-\beta_0 X_k} \quad (9)$$

$$f(X_k|H_1) = \beta_1 e^{-\beta_1 X_k} \quad (10)$$

where $X_k \geq 0$, and without loss of generality, we assume that $0 < \beta_0 < \beta_1$.

ρ	η (algorithm)	η (exhaustive search)
.1	1.2816	1.2816
.2	0.8596	0.8596
.4	0.8596	0.8596
.6	0.8596	0.8596
1	0.8596	0.8596

Table 1: Local observation thresholds obtained for Gaussian observations.

ρ	η (algorithm)	η (exhaustive search)
.1	0.0702	0.0702
.2	0.1488	0.1488
.4	0.3406	0.3406
.6	0.3582	0.3584
1	0.3582	0.3584

Table 2: Local observation thresholds obtained for exponential observations.

Similarly, the LR threshold τ can be translated directly to the local observation threshold in the exponential observations case. The local decision rule is in the form of the following:

$$\begin{array}{ccc} u_k = 0 \\ X_k \gtrless \eta \\ u_k = 1 \end{array} \quad (11)$$

Consequently, the false alarm rate p_f and the detection probability p_d at local sensors can be expresses as

$$p_f = 1 - e^{-\beta_0 \eta} \quad (12)$$

$$p_d = 1 - e^{-\beta_1 \eta} \quad (13)$$

It can be shown that the rate $R = p_f$ is a monotonically increasing function of η . Hence, we can adopt the modified algorithm to obtain the optimal threshold η numerically. The simulation results for different rate constraint ρ are listed in Table 2 at channel SNR = 0dB.

The same observations hold as what were observed in the Gaussian observations example: as ρ increases, the obtained threshold will eventually remain unchanged. The results obtained using the proposed algorithm and through exhaustive search in Table 2 for the exponential observation case match very well for different ρ 's.

Under exponential observations, similarly, as shown from the plot of the KL distance versus the rate constraint ρ in Fig. 3, for small ρ , increasing ρ improves $D(P_0||P_1)$ yet as ρ becomes large, there's a "saturation" effect for $D(P_0||P_1)$.

5 Conclusion

In this paper, under Neyman-Person framework we have developed a procedure to obtain the optimal local sensor quantization threshold for a binary hypothesis testing problem under communication constraint. In particular we consider the asymptotic case where the number of sensors is large. By adopting energy detector as the fusion statistic and applying central limit

theorem, we pose the design problem as maximizing the Kullback-Leibler distance of the distributions of the fusion statistic under both hypothesis while subject to certain rate constraint. Efficient algorithms are developed to numerically obtain the optimal threshold at local sensors for different scenarios of the rate function.

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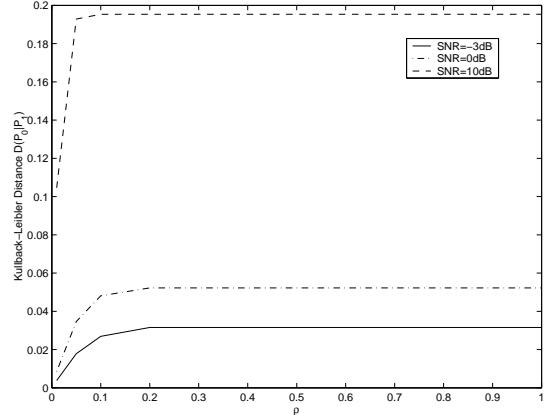


Figure 2: Kullback-Leibler distance as a function of ρ at different channel SNRs under Gaussian observations.

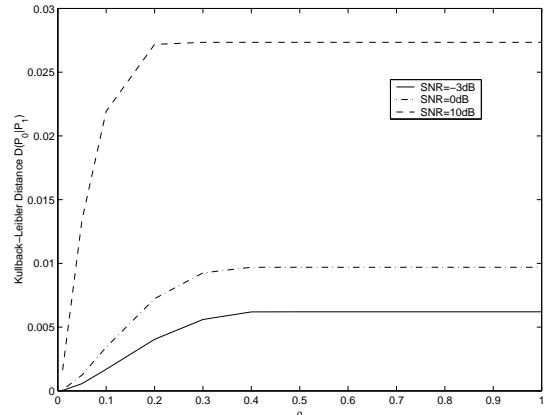


Figure 3: Kullback-Leibler distance as a function of ρ at different channel SNRs under exponential observations.